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1986 J. Phys. A: Math. Gen. 19 563

(http://iopscience.iop.org/0305-4470/19/4/017)

J. Phys. A: Math. Gen. 19 (1986) 563-574. Printed in Great Britain

Hamiltonian studies of the two-dimensional *n*-component cubic model: I

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Received 2 January 1985, in final form 7 June 1985

Abstract. The phase diagram and the critical properties of the Hamiltonian version of the two-dimensional n-component cubic model are investigated. Judging from the results of simple limits, mean-field calculation and RG transformations, the phase diagram of the one-dimensional quantum system is similar to that of the two-dimensional classical model. Several RG transformations were used to investigate the critical properties using different cell sizes in the transformation. The coincidence of critical and tricritical fixed points and the presence of a marginal operator showed the formation of the Ashkin-Teller fixed line and the breaking of the universality. The cubic transition is found to be first order for $n > 2$.

1. Introduction

The n-component cubic model represents a very general type of discrete lattice model and includes, in special cases, the Potts model (Potts 1952) and the Ashkin-Teller model (Ashkin and Teller 1943). Because of this the model shows many extraordinary aspects of critical behaviour. The nature of the phase transition changes with increasing number of spin components; as a result of competition the model is possessed of a multicritical point whose type also depends on n; in two dimensions (2D) for $n=2$ there is a critical line in the system (the description of this line of fixed points by the usual renormalisation group (RG) transformation has not led to satisfactory results). However, by increasing the space of parameters by taking n to be a variable, and then extrapolating to $n=2$, a satisfactory description is obtained (Nienhuis *et al* 1983).

The n-component cubic model was originally introduced as a means of modelling anisotropic magnetic systems (Kim *et al* 1975, Aharony 1977), but it has many other applications. In 2D, for example, the model describes some phase transitions of absorbed monolayers (for a review see Schick (1983)). The phase diagram of the model in 2D was determined by Domany and Riedel (1979), and the critical properties were investigated by Nienhuis *et al* (1983). In the latter paper the order of the phase transition was determined for different values of n by using the vacancy generating RG transformation (Nienhuis *et al* 1979). Furthermore, by using an exact mapping onto a solid-on-solid model for the $n=2$ case, new interconnections were obtained among the class of cubic, Potts and Ashkin-Teller phase transition phenomena.

In this paper the phase transition properties of the Hamiltonian version of the 2D model are investigated. By taking the time-continuum limit (Kogut 1979), the 2D model is mapped onto a 1D quantum problem. In most cases the anisotropy in the 2D models

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0305-4470/86/040563+12$02.50

is irrelevant to the critical properties, and many approximate treatments are easier to apply in 1D.

The phase diagram of the system is determined by investigating simple limiting cases, by mean-field calculations, and by RG transformation. The critical properties of the system are investigated by several RG transformations. We use block transformation and decimation transformation and apply different sizes of the cell in the transformation, which together enable us to extrapolate the results.

The paper is arranged as follows: § 2 contains the time-continuum limit, the different representations of the problem and the analysis of different limiting cases. In § 3 the phase diagrams of the system obtained by mean-field and RG methods are presented, and in § 4 the critical properties of the model are investigated by RG transformations. Section 5 presents a short discussion. Some technical points of this paper may be found in detail in a preprint, available from the author (Iglói 1984).

2. Formalism

In the n-component cubic model there are spins on a lattice, each spin with $2n$ states, denoted by $|1\rangle, |2\rangle, \ldots, |2n\rangle$. The energy of the system depends on the nearest-neighbour configuration. The interaction energy between neighbouring spins is

$$H_\lambda |s_i s_{i+1}\rangle = \begin{cases} -\frac{1}{2}\lambda_1 |s_i s_{i+1}\rangle, & \text{if } s_i = s_{i+1}, \\ +\frac{1}{2}\lambda_1 |s_i s_{i+1}\rangle, & \text{if } s_i = s_{i+1} + n, \\ \frac{1}{2}\lambda_2 |s_i s_{i+1}\rangle, & \text{otherwise.} \end{cases} \tag{2.1}$$

For $\lambda_1 = \lambda_2$, the model reduces to the $2n$-state Potts model (Potts 1952), while for $n = 2$ it corresponds to the Ashkin–Teller model (Ashkin and Teller 1943).

The Hamiltonian version of the 2D n-component cubic model is a 1D chain with the classical (2.1) interaction in the presence of an external spin-flip field, which depends on two parameters h_1 and h_2:

$$\langle s_i | H_h | s_j' \rangle = \begin{cases} \delta_{ij} h_2, & \text{if } s_i = s_j', \\ -\delta_{ij} h_2, & \text{if } s_i = s_j' + n, \\ -\delta_{ij} h_1, & \text{otherwise.} \end{cases} \tag{2.2}$$

In this representation the coupling part of the Hamiltonian (2.1) is diagonal. Therefore it may be called the strong-coupling (or low temperature) representation. For the $h_1, h_2 \to 0$ limit, in the ground state all spins are in the same state which results in a $2n$-fold degeneracy.

In the weak-coupling limit it is more convenient to use the representation where the spin-flip field is diagonal. Let us introduce the following orthonormal set of vectors:

$$\begin{aligned} |1'\rangle &= (1/\sqrt{2n})\{|1\rangle + |2\rangle + \ldots + |2n\rangle\}, \\ |2'\rangle &= (1/\sqrt{2n})\{|1\rangle + \omega|2\rangle + \ldots + \omega^{2n-1}|2n\rangle\}, \\ &\vdots \\ |(2n)'\rangle &= (1/\sqrt{2n})\{|1\rangle + \omega^{2n-1}|2\rangle + \ldots + \omega^{(2n-1)^2}|2n\rangle\}, \end{aligned} \tag{2.3}$$

where $\omega = \exp(2\pi i/2n)$.

In this representation H_λ leaves invariant the sum of the spins along the chain, modulo $2n$. Therefore the eigenstates of the chain belong to $2n$ disjoint sectors that can be characterised by the functions $|1'1'\ldots 1'\rangle$, $|2'1'\ldots 1'\rangle, \ldots, |(2n)'1'\ldots 1'\rangle$, and will be called the 1st, the 2nd and the $2n$th subspace, respectively. From symmetry considerations it follows that the 1st subspace is non-degenerate, while the 2nd, 4th, ..., $2n$th subspaces are n-fold degenerate, and the 3rd, 5th, 7th, ..., $(2n-1)$th subspaces are $(n-1)$-fold degenerate. At the thermodynamic limit, when the length of the chain tends to infinity, further degeneracies may occur. The type and degeneracy of the ground state characterise the different phases. We shall return later to this question.

Besides the weak- and strong-coupling regions, characterised by the conditions $h_1, h_2 \gg \lambda_1, \lambda_2$ and $\lambda_1, \lambda_2 \gg h_1, h_2$ respectively, there exists another simple limiting case, when $\lambda_2, h_2 \gg \lambda_1, h_1$.

Let us now make use of the following combination of the states:

$$\begin{aligned}
|1''\rangle &= \frac{1}{\sqrt{2}}(|1\rangle + |n+1\rangle), & |(n+1)''\rangle &= \frac{1}{\sqrt{2}}(|1\rangle - |n+1\rangle),\\
|2''\rangle &= \frac{1}{\sqrt{2}}(|2\rangle + |n+2\rangle), & |(n+2)''\rangle &= \frac{1}{\sqrt{2}}(|2\rangle - |n+2\rangle),\\
&\vdots\\
|n''\rangle &= \frac{1}{\sqrt{2}}(|n\rangle + |2n\rangle), & |(2n)''\rangle &= \frac{1}{\sqrt{2}}(|n\rangle - |2n\rangle).
\end{aligned} \tag{2.4}$$

Now it is easy to see that the part of the Hamiltonian proportional to h_2 and λ_2 is diagonal. The ground state of the system at the limit $\lambda_2, h_2 \gg \lambda_1, h_1$ is n-fold degenerate: $|1''1''\ldots 1''\rangle, |2''2''\ldots 2''\rangle, \ldots, |n''n''\ldots n''\rangle$. The relevant excitations have a simple form either for $h_1 \ll \lambda_1$ or when $\lambda_1 \ll h_1$. In the first case the system can be represented by an Ising-like state; each spin can be in two possible states, say $|1''\rangle$ and $|(n+1)''\rangle$. The form of the Hamiltonian reduces to

$$H = -\tfrac{1}{2}\lambda_1 \sum_i \sigma_i^x \sigma_{i+1}^x - h_2 \sum_i (\sigma_i^z - 1) + \mathrm{O}(h_1/\lambda_2) \tag{2.5}$$

where σ_i^x, σ_i^z Pauli operators act on the states $|1''\rangle$ and $|(n+1)''\rangle$. At this limit the phase transition is Ising-like and takes place at

$$h_2/\lambda_1 = \tfrac{1}{2}. \tag{2.6}$$

At the other limit, for $\lambda_1 \ll h_1$, the relevant excitations are, at each site, a combination of the states $|1''\rangle, |2''\rangle, \ldots, |n''\rangle$. The Hamiltonian can be written as

$$H = -\tfrac{1}{2}\lambda_2 \sum_i (\delta_{s_i s_{i+1}} - 1) - 2h_1 \sum_i \sum_{k=1}^{n} M_i^k + \mathrm{O}(\lambda_1/h_2) \tag{2.7}$$

where $s_i = 1, 2, \ldots, n$. M is an $n \times n$ matrix

$$M = \begin{bmatrix} 0 & 0 & \ldots & 1 \\ 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & & 1 & \vdots \\ 0 & 0 & \ldots 1 & 0 \end{bmatrix}$$

acting on the states $|1''\rangle, |2''\rangle, \ldots, |n''\rangle$. The Hamiltonian (2.7) describes the n-state Potts model. From the exact solution of Baxter (1973) it is known that the phase transition takes place at

$$4nh_1/\lambda_2 = 1. \tag{2.8}$$

Furthermore it is first order for $n > 4$, and continuous for $n \leqslant 4$. In closing the section let us briefly summarise the possible phases of the system. For positive values of the couplings the system exhibits three phases. If the couplings are much smaller than the external fields ($\lambda_1, \lambda_2 \ll h_1, h_2$) the system is paramagnetic, and the ground state is non-degenerate. (In the figures this region is denoted by I.) At the opposite limit, when the external fields are much weaker than the couplings ($h_1, h_2 \ll \lambda_1, \lambda_2$), the system is ferromagnetically ordered and the ground state is $2n$-fold degenerate (denoted by III). Finally, at the $\lambda_2, h_2 \gg \lambda_1, h_1$ limit, the system is partially ordered, and the ground state is n-fold degenerate (denoted by II).

On the phase boundaries of the system the following can be determined. At the $2n$-state Potts point ($\lambda_1 = \lambda_2$), and for $\lambda_1 < \lambda_2$, there is no partially ordered phase. By increasing the strength of the external fields, the system goes in one step from the ferromagnetic phase to the paramagnetic phase. This is the cubic transition. By increasing the value of λ_2 the partially ordered phase appears and the cubic transition line bifurcates. At the $\lambda_2 \gg \lambda_1$ limit, according to equation (2.5), the transition between the ferromagnetic and the partially ordered phase is equivalent to the transition in the Ising model. On the other hand, the transition between the partially ordered and the paramagnetic phase can be described by the n-state Potts model (equation (2.7)).

3. The phase diagram

The phase diagram of the system is determined by mean-field calculation and by the RG method.

First the result of the mean-field calculation is presented. The simplest form is chosen for the trial wavefunction; this is the product of the one-spin wavefunctions (Iglói 1984).

The mean-field phase diagram is drawn in figure 1 for $n = 3$. The topology of the phase diagram is the same for all values of n, and is in accordance with those written

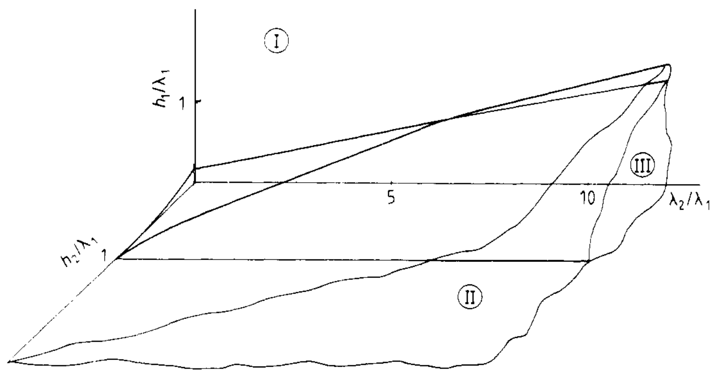

Figure 1. Mean-field phase diagram for $n = 3$. The model is paramagnetic in region I, partially ordered in region II and ferromagnetic in region III.

in § 2. The order of the phase transitions obtained is different for $n=2$ and for $n \geq 3$. The types of transitions between the different phases for $n=2$ and for $n \geq 3$ are given in table 1.

We have also determined the phase diagram in the $n \to \infty$ limit, since the mean-field phase diagram is generally exact, if the number of components of the spin or the range of interactions goes to infinity. The phase diagram for $n \to \infty$ is shown in figure 2 in the following plane:

$$\begin{aligned} &\lambda_1/2n = 1, \\ &h = 2nh_1/\lambda_1 = 2nh_2/\lambda_2, \qquad (3.1) \\ &\lambda = \lambda_2/\lambda_1 = h_2/h_1. \end{aligned}$$

We will touch upon the importance of this subspace in the following paper (Iglói 1986, hereafter referred to as paper II). Here we use it for better representation.

The mean-field phase diagram is exact for the cubic transition line, as we can state in comparing it to the result of the $1/n$ expansion (paper II). A further possibility to check this is to compare the phase diagram with the asymptotic transition lines (equations (2.5) and (2.7)). In this way the paramagnetic partially ordered transition coincides with equation (2.7) whereas the partially ordered ferromagnetic transition does not agree with the asymptotic line (2.5).

Table 1. Nature of transitions in the mean-field calculation. I, II and III denote the same regions as in figure 1.

Transition	n	Type of transition
I → III	2	2nd order, 1st order, tricritical points
	≥ 3	1st order
I → II	2	2nd order
	≥ 3	1st order
II → III	2	2nd order
	≥ 3	2nd order

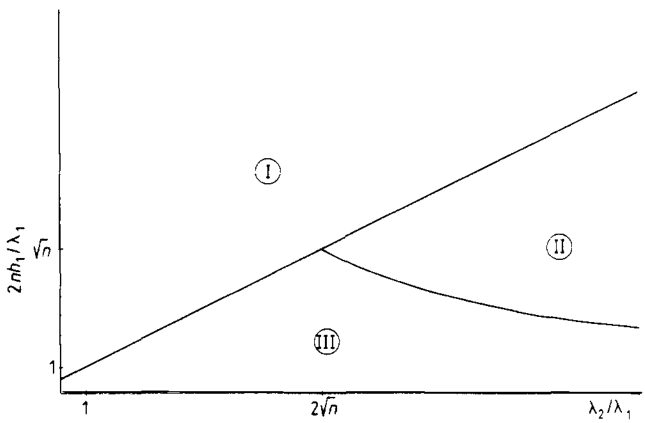

Figure 2. Mean-field phase diagram for $n \to \infty$. The II → III transition takes place on the line $h_1/\lambda_1 = (\lambda_2/\lambda_1)^{-1}$.

To determine the phase diagram by the RG method a self-dual decimation transformation (ST) is used, which was proposed by Fernandez-Pacheco (1979) for the Ising model, and was later applied to different problems (Horn *et al* 1980, Hu 1980, Sólyom and Pfeuty 1981, Iglói and Sólyom 1983a, 1984). The transformation generally gives the exact phase-transition points for self-dual models, but it is also quite accurate for non-self-dual models for the phase diagram. The derivation of the RG transformation as well as the structure of the fixed points is like those obtained for the Ashkin-Teller model (Iglói and Sólyom 1984). The RG transformation does not generate new couplings, and the recursion relations have three types of trivial fixed point solution yielding three different phases. The region of attraction of these fixed points is shown in figure 3 for $n=2$. It is mentioned that the structure of the phase diagram obtained by this

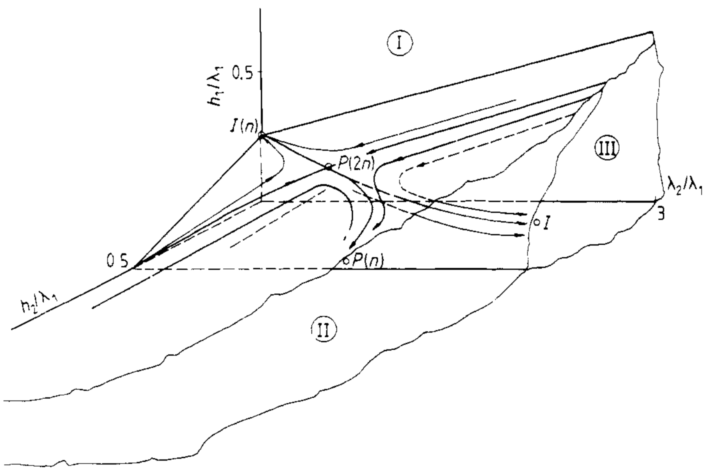

Figure 3. Phase diagram for $n=2$ obtained by ST. $I(n)$, $P(2n)$, $P(n)$ and I denote the non-trivial fixed points.

transformation does not depend on the value of n; it is only that the boundaries of the phases are changing.

The different phases can be characterised as follows:

I, paramagnetic phase: the couplings scale to $h_1 =$ arbitrary, $h_2 =$ arbitrary, $\lambda_1 = 0$, $\lambda_2 = 0$,

II, partially ordered phase: the couplings scale to $h_1 = 0$, $h_2 =$ arbitrary, $\lambda_1 = 0$, $\lambda_2 =$ arbitrary,

III, ferromagnetically ordered phase: the couplings scale to $h_1 = 0$, $h_2 = 0$, $\lambda_1 =$ arbitrary, $\lambda_2 =$ arbitrary.

The critical surfaces, which separate the different phases, are characterised by the following non-trivial fixed points.

(i) The points of the surface separating the paramagnetic and the ferromagnetic region scale to $h_1 = 0$, $h_2 = 0$, $\lambda_1 = 0$, $\lambda_2 = 0$, with, however,

$$\frac{2nh_1}{\lambda_1} = \text{finite}, \qquad \frac{h_2}{\lambda_1} = 0, \qquad \frac{\lambda_2}{\lambda_1} = 0.$$

This fixed point is denoted by $I(n)$ in figure 3. (The notation is explained by the fact that for $n=2$ it is an Ising-like fixed point.) The position of the fixed point as well as the eigenvalues of the linearised RG transformation at the fixed point are given in table 2.

(ii) The points of the critical surface separating the paramagnetic and the partially ordered region scale to $h_1=0$, $\lambda_1=0$, $\lambda_2=0$, $h_2=$ arbitrary, with, however,

$$2nh_1/\lambda_2=\tfrac{1}{2}, \qquad \lambda_1/\lambda_2=0.$$

This is the fixed point of the n-state Potts model, denoted by $P(n)$. The thermal eigenvalue is $(n+2\sqrt{n}+2)/(\sqrt{n}+2)$.

Table 2. Position of the $I(n)$ fixed point and the eigenvalues of the ST transformation for different values of n.

n	$2nh_1/\lambda_1$	Λ_1^{T}	Λ_2^{T}	Λ_3^{T}
2	1	2	0.707	0.707
3	0.733	2.476	0.814	0.737
4	0.650	2.870	0.861	0.761
5	0.611	3.212	0.888	0.779
10	0.547	4.532	0.942	0.829
100	0.504	14.214	0.993	0.936
∞	$0.5+8/2n$	$\sqrt{2n}$	1	1

(iii) The points of the surface separating the ferromagnetic region and the partially ordered region scale to $h_1=0$, $h_2=0$, $\lambda_1=0$, $\lambda_2=$ arbitrary, with, however,

$$h_1/\lambda_1=0, \qquad h_2/\lambda_1=\tfrac{1}{2}.$$

This is an Ising-like fixed point denoted by I; the thermal eigenvalue is 2 and the critical exponent ν is 1.

(iv) The points of the line where the three phases coexist scale to $h_1=0$, $h_2=0$, $\lambda_1=0$, $\lambda_2=0$, with, however,

$$\frac{2nh_1}{\lambda_1}=1, \qquad \frac{2nh_2}{\lambda_1}=1, \qquad \frac{\lambda_2}{\lambda_1}=1.$$

This is the critical point of the $2n$-state Potts model, denoted by $P(2n)$. The eigenvalues at this point are

$$\Lambda_1^{\mathrm{T}}=(2n+2\sqrt{2n}+2)/(\sqrt{2n}+2),$$
$$\Lambda_2^{\mathrm{T}}=1+2/(\sqrt{2n}+1)(1+\tfrac{1}{2}\sqrt{2n}),$$
$$\Lambda_3^{\mathrm{T}}=1.$$

This renormalisation group procedure accurately describes the phase boundaries in the region between the $I(n)$ and $P(2n)$ fixed points, as is shown in paper II by comparing the results with series expansion. However, in some respects it gives a rather crude description. First of all, the phase transitions are determined to be second order for any values of n, in contrast to the fact that transitions described by the $I(n)$, $P(2n)$, $P(n)$ fixed points should be of first order for large values of n. Otherwise the bifurcation line always goes through the $2n$-state Potts point, although it should move to higher values of λ_2 and h_2, as is shown by series expansion results (paper II).

4. Critical properties

The critical properties of the model are determined by the block transformation (BT) and by the usual decimation transformation (DT). (For a review of the quantum RG methods see Pfeuty *et al* (1982).) These two transformations are duals of each other for self-dual models (Sólyom 1981); in our case they supply different, but rather similar, information. In both transformations starting with the Hamiltonian with four couplings, the RG transformation generates three new ones, but further renormalisation steps do not increase the number of couplings. In both methods, in order to make extrapolations, we use different sizes of the cells in the transformation.

A schematic picture of the RG phase diagram containing the qualitative features of the results of the two transformations is given in figure 4 for different values of n in the extended space of couplings. The third axis, pointing perpendicularly to the physical plane, serves to represent all new couplings, and therefore somehow plays the role of the dilution in the 2D classical model.

The phase diagram consists of the three already known phases, which are characterised by similar trivial fixed points as in ST, discussed in § 3. Otherwise, the transition between the ferromagnetic and partially ordered regions in BT and DT remains Ising-like, and is controlled by the one-fold unstable I fixed point. The three other transitions, however, as one can see in figure 4, are controlled by critical, tricritical and discontinuity fixed points, denoted by the superscripts 'c', 't' and 'd', respectively. The $I^{c}(n)$, $I^{d}(n)$, $P^{c}(n)$, $P^{d}(n)$ fixed points are one-fold unstable, the $I^{t}(n)$, $P^{t}(n)$, $P^{c}(2n)$, $P^{d}(2n)$ are two-fold unstable, while the $P^{t}(2n)$ fixed point is three-fold unstable. In all cases the critical fixed points control the second-order transitions of the physical model, the discontinuity fixed points describe first-order transitions, while the tricritical fixed points separate the second-order and first-order transition regions. (In the simplest case, when two spins are in the cell in the transformation, there are other non-trivial fixed points, which are moving on the line between the $I(n)$ and $P(2n)$ fixed points, and the degree of instability of the different fixed points depends on the value of n. Since by using larger cells in the transformation these moving fixed points do not appear, their appearance for the simplest case is attributed to the effect of approximation.)

The evaluation of the structure of the $I(n)$, $P(n)$ and $P(2n)$ fixed points with increasing value of n is the well known process characteristic for the crossover from second-order to first-order transitions (Nienhuis *et al* 1979, Sólyom and Pfeuty 1981). The critical and tricritical fixed points move towards each other. At a critical value (denoted by n_c^I and n_c^P for the cubic and the $2n$-state Potts transitions, respectively) they coincide, the next-to-leading eigenvalue of the linearised RG equations at the fixed point is exactly 1, and the crossover eigenvalue is very close to 1. If the value of n is further increased, the two fixed points annihilate each other, and the transition in the physical system is controlled by the discontinuity fixed point, i.e. it is of first order.

The critical and tricritical thermal exponents as a function of n in the $I(n)$ and $P(2n)$ fixed points for DT are given in figure 5 and figure 6, respectively. In figure 6 the conjectured values (den Nijs 1979) are also drawn. The calculated exponents even for $B = 5$ are rather far from the conjectured ones, but an extrapolation procedure gives satisfactory results (Pfeuty *et al* 1982, Iglói and Sólyom 1983b).

The critical values of n and the crossover eigenvalues for the different transitions and different sizes of the cells are given in table 3. One can see that the n_c^I values are larger than two, and they monotonically decrease. This series presumably tends to

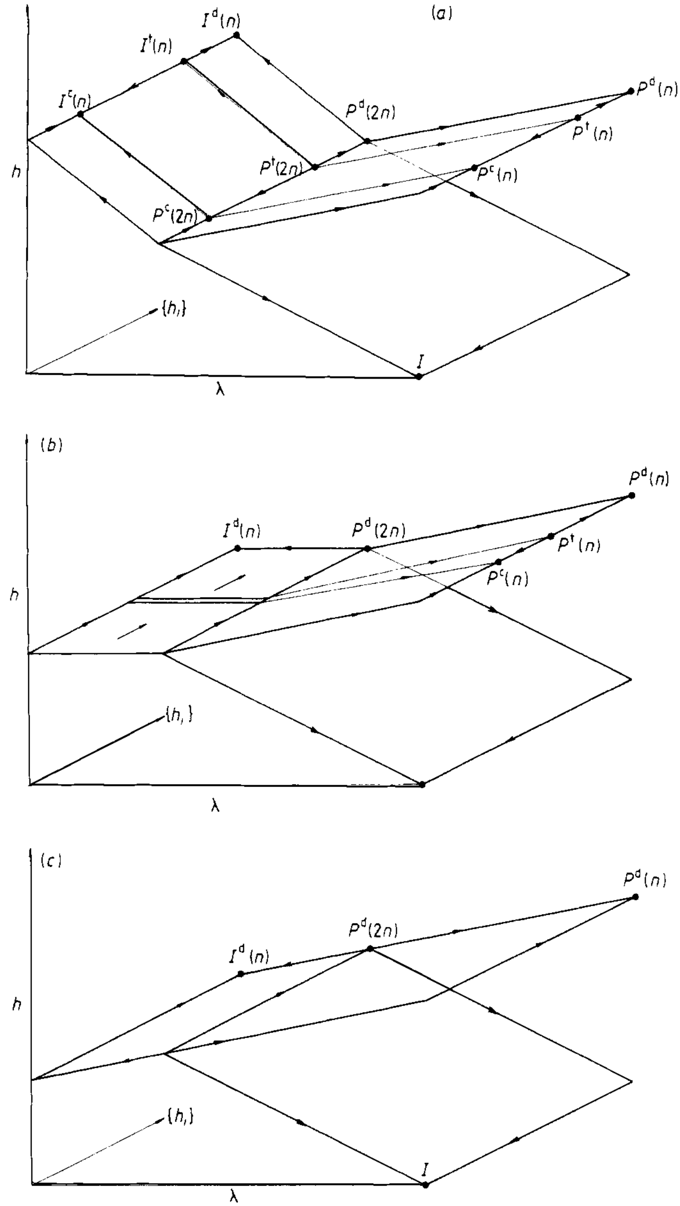

Figure 4. Schematic renormalisation group phase diagrams obtained by decimation transformation and by block transformation. The $\{h_i\}$ axis serves to represent all new couplings generated by the transformations. (a) $n<2$, (b) $n=2$ (Ashkin-Teller model), (c) $n>4$. Dots represent non-trivial fixed points; the double line for $n=2$ is the Ashkin-Teller fixed line.

two, which is supported by the fact that at $n=2$ the transition is of second order, and the inequality $n_c^I < n_c^P$ holds for all calculated sizes of the cell in the transformation and n_c^P tends to 2. The crossover eigenvalues given in table 3 are very close to 1, both in the $I(n)$ and in the $P(2n)$ fixed points at the annihilation values of n. These

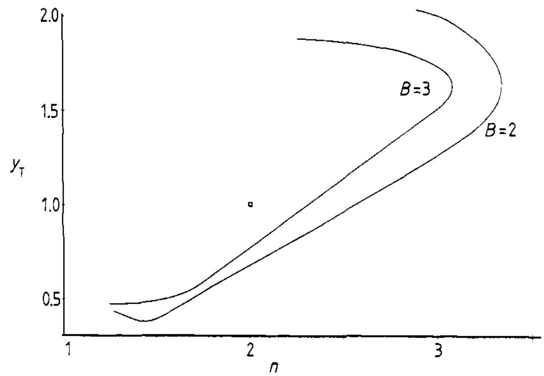

Figure 5. Critical and tricritical thermal exponents at the $I(n)$ fixed point calculated by decimation transformation for $B=2$ and 3. The square denotes the exactly known value for the Ising model.

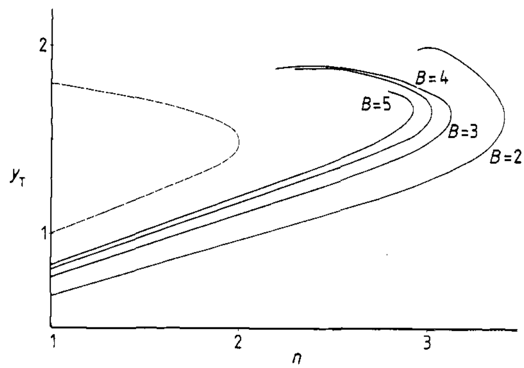

Figure 6. Critical and tricritical thermal exponents at the $2n$-state Potts point by using different sizes of the cell in the transformation. The exact values (den Nijs 1979) are represented by the broken line.

Table 3. Critical values of n, where the critical and tricritical fixed points annihilate each other. Superscripts I and P refer to the $I(n)$ and $P(2n)$ fixed points, respectively; n_c^I(BT) and n_c^I(DT) were calculated by block transformation and by decimation transformation, respectively. The values in parentheses are the next-to-leading eigenvalues of the RG transformations when the critical and tricritical fixed points coincide.

B	n_c^I(BT)	n_c^I(DT)	n_c^P
2	3.24 (1.009)	3.34 (1.010)	3.41 (0.993)
3	3.04 (0.972)	3.08 (0.980)	3.12 (1.014)
4		2.99	3.02 (1.040)
5			2.92
∞	2 (1)	2 (1)	2 (1)

facts signal the presence of a marginal operator and a line of fixed points with continuously varying critical exponents, i.e. the Ashkin–Teller fixed line.

Thus we can conclude that our findings are in accordance with the assumption that the breaking of universality on the cubic transition line takes place in two steps for increasing value of n:

(i) At $n=2$ the critical indices of the second-order transition depend on the coupling.

(ii) For $n>2$ the transition is of first order, and the latent heat depends on the coupling.

Finally we would mention that the neighbourhood of the multicritical point cannot be investigated satisfactorily by these RG transformations for $n>2$. With these calculations, similar to the ST, the multicritical point and the $2n$-state Potts point are found to be the same, which is in contrast to the result of the $1/n$ expansion (paper II). The nature of the multicritical point is still unsolved, even for the 2D classical model.

5. Summary

In this paper the phase diagram and the critical properties of the Hamiltonian version of the 2D n-component cubic model were determined by different methods. Analysis of simple limiting cases and the results of mean-field calculation and RG transformations have shown that the phase diagram of the (1+1)D model is similar to that of the 2D model.

The critical properties of the model were investigated by different RG transformations. By using different sizes of the cell in the transformation, it was possible to extrapolate the results. The calculation gave an account of the line of fixed points in the Ashkin–Teller model, and of the first-order transition on the cubic transition line for $n>2$.

Comparing our results with those obtained by Nienhuis *et al* (1983) on the n-component cubic lattice gas, some qualitative differences may be obtained. First, the positions of the fixed points of the two transformations governing the cubic critical behaviour are not equivalent to each other in the time-continuum limit. Furthermore, according to Nienhuis *et al* (1983) the cubic tricritical and the first-order behaviour is governed by the $2n$-state Potts tricritical and discontinuity fixed points, and the cubic transition is of the $O(n)$ universality class. This latter statement is in accordance with the finite size scaling results of Blöte and Nightingale (1984). In our case there are separate tricritical ($I^t(n)$) and discontinuity ($I^d(n)$) fixed points controlling the cubic transition, and also the merging of these fixed points takes place. These differences may probably be ascribed to the fact that the generalisations of the model by the two RG transformations are different.

Acknowledgment

The author is indebted to J Sólyom for useful discussions, and he is also grateful to the referee for some useful recommendations.

References

Aharony A 1977 *J. Phys. A: Math. Gen.* **10** 389
Ashkin J and Teller E 1943 *Phys. Rev.* **64** 178
Baxter R J 1973 *J. Phys. C: Solid State Phys.* **6** L445
Blöte H W J and Nightingale M P 1984 *Physica* **129A** 1
Domany E and Riedel E K 1979 *Phys. Rev.* B **19** 5817
Fernandez-Pacheco A 1979 *Phys. Rev.* B **19** 3173

Horn D, Karliner M and Jankielowicz S 1980 *Nucl. Phys.* B **170** (FS1) 467
Hu B 1980 *J. Phys. A: Math. Gen.* **13** L321
Iglói F 1984 *Preprint KFKI-1984-128, 129*
—— 1986 *J. Phys. A: Math. Gen.* **19** 577
Iglói F and Sólyom J 1983a *Phys. Rev.* B **28** 2785
—— 1983b *Phys. Rev.* B **28** 2792
—— 1984 *J. Phys. A: Math. Gen.* **17** 1531
Kim D, Levy P M and Uffer L F 1975 *Phys. Rev.* B **12** 989
Kogut J 1979 *Rev. Mod. Phys.* **51** 659
Nienhuis B, Berker A N, Riedel E K and Schick M 1979 *Phys. Rev. Lett.* **43** 737
Nienhuis B, Reidel E K and Schick M 1983 *Phys. Rev.* B **27** 5625
den Nijs M P M 1979 *J. Phys. A: Math. Gen.* **12** 1857
Pfeuty P, Jullien R and Penson K A 1982 *Real Space Renormalisation* ed T W Burkhardt and J M J van Leeuwen (Berlin: Springer) p 119
Potts R B 1952 *Proc. Camb. Phil. Soc.* **48** 106
Schick M 1983 *Surf. Sci.* **125** 94
Sólyom J 1981 *Phys. Rev.* B **24** 230
Sólyom J and Pfeuty P 1981 *Phys. Rev.* B **24** 218